A NEW SPECTRAL FEATURE FOR SHAPE COMPARISON

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ABSTRACT

Boundary is an important characteristic to describe a shape in computer vision and image retrieval. A normalized Canonical Polygonal Representation (NCPR) which represents uniquely a polygonal boundary was introduced in the previous study. The objective of this study is to show 1) how a shape feature can be obtained from NCPR, 2) how to utilize this shape feature in shape comparison, 3) to explore effects of the feature parameters on shape comparison, and 4) to explore robustness and closeness of some similarity measures to visual similarity while using the feature proposed. Experimental results show that the approach proposed is robust and discriminative among similar and dissimilar shapes.

Keywords — Image shape analysis, Image coding, Image classification, Pattern recognition, Image matching

1. INTRODUCTION

Image retrieval is a growing area of research due to the huge amount of image data that has been produced and archived. Effective image representation technique is crucial for retrieval performance. Shape comparison is an essential part of the retrieval process while querying similar images. As an “imject” (object in an image) description, shape is an important element. Shapes can be represented by different forms such as a signature function, skeleton or curvature scale space. In this study, we employ a shape feature obtained from the NCPR which is a string representation of a polygonal boundary.

In representation, uniqueness and geometric invariance are important characteristics to distinguish shapes under rigid deformations at least. In practice, shapes compared may not have the same number of vertices. So a shape descriptor should be able to accommodate this condition. An NCPR string is unique for a polygon and is also invariant under rigid transformation (that is translation, scale and rotation invariant), and it can be used to match shapes with a different number of vertices by the feature vector proposed.

As a spectral descriptor, the Fourier coefficients can be employed in shape representation which can be obtained by computing discrete Fourier coefficients. Performance of the Fourier descriptor is acknowledged in [2].

The outline of this paper is as follows: Section 2 describes how the shape function \( y[x_i] \) is obtained from \( P_N \) and how the feature vector \( f \) is computed from \( y[x_i] \); section 2.3 defines distance/similarity measures. Section 3 explains the experimental setup and section 4 presents a discussion of
the results. As a convention, a bold letter represents a vector or an array, a box bracket ‘[ ]’ refers to a discrete value when used with a function or a vector, and abbreviations AVR and ‘s’ stand for ‘average’ and ‘similarity’, respectively, in figures.

2. METHOD

2.1. Shape Representation

In this section we describe how to derive a shape function which is used to obtain the shape feature proposed from the canonical representation of a polygonal boundary. We propose three methods to derive a descriptive function for the shape. After obtaining the function, an interpolation is necessary to obtain a normalized axis with equal intervals for the Fourier transform. The shape function derivation methods proposed are:

1. Values for the dependent variable y are obtained from the set of \( \delta_i \) and that of independent variable \( x \) are obtained from the set of \( \delta_{2i} \), formally:
   \[
   y\{x_i\} = \delta_{1i} \quad (2.1)
   \]
   where \( x_i = c \left( \sum_{j=1}^{i} \delta_{2j} - \delta_{2i} \right) \) for \( i = 1, 2, ..., n \).

2. \( y \) is a combination of the sequence \( \{\delta_i\} \) followed by the sequence \( \{\delta_{2i}\} \), and \( x \) is obtained from indexes, formally:
   \[
   y\{x_i\} = \begin{cases} 
   \delta_{1i} & \text{for } i = 1, 2, ..., n \\
   \delta_{2i} & \text{for } i = n + 1, n + 2, ..., 2n 
   \end{cases} 
   \quad (2.2)
   \]
   where \( x_i = c \left( i - 1 \right) \), for \( i = 1, 2, ..., 2n \).

3. Odd indexed \( y \) values are obtained from \( \delta_i \) and even indexed \( x \) values are obtained from \( \delta_{2i} \), formally:
   \[
   y\{x_i\} = \begin{cases} 
   \delta_{1i} & \text{for } i = 1, 3, ..., 2n - 1 \\
   \delta_{2i} & \text{for } i = 2, 4, ..., 2n 
   \end{cases} 
   \quad (2.3)
   \]
   where \( x_i = c \left( i - 1 \right) \), for \( i = 1, 2, ..., 2n \).

The algorithm to obtain NCPR shape function:

Input: NCPR \( P_N \).
Output: Shape function \( y[x] \).
1. Obtain shape function \( \hat{y}[x] \) using (2.1) - (2.3) with scale coefficient \( c > 0 \).
2. Apply linear interpolation to \( \hat{y} \) to obtain \( y[x] \).

2.2. Feature Vector

A feature vector is a characteristic representation of a function. In this study, the Fourier coefficients are used as a feature vector due to its discriminative property. The feature vector \( f \) is defined as
   \[
   f = \{Y[i] | i = 1, 2, ..., K\} \quad (2.4)
   \]
   where \( Y \) is the discrete Fourier transform of \( y[x] \) and \( K \) is size of \( f \).

2.3. Distance Measures

Images in a database can be compared by means of distance measures which are used in the computation of a similarity score between a query and some reference images. We employed two types of similarity measures. The first group given in (2.5) and (2.6) is a weighted average of the angle and magnitude distances while the second group given in (2.7) and (2.8) measures the Euclidean distance between the vectors compared. The similarity measures utilized in this study are:

\[
S_1 = c_a (1 - D_{a}) + c_d (1 - D_{d1}) 
\]
\[
S_2 = c_a (1 - D_a^b) + c_d (1 - D_{d1}^k) 
\]
\[
S_3 = 1 - D_{d2} 
\]
\[
S_4 = 1 - D_{d2}^k 
\]

where
\[
D_a = \frac{\alpha}{90} 
\]
\[
D_{d1} = \frac{| ||q|| - ||r|| |}{||q|| + ||r||} 
\]
\[
D_{d2} = \frac{||q-r||}{||q|| + ||r||} 
\]
where weighting coefficients \( c_a, c_d > 0 \) and \( c_a + c_d = 1 \), distance functions \( D_a, D_{d1}, \) and \( D_{d2} \) are \([0,1]^n \rightarrow [0,1], \alpha \in [0^\circ, 90^\circ] \) is the angle in degrees between feature vectors \( q, r \in [0,1]^n \), constant \( k = 2 \), \( \|*\| \) refers to Euclidean norm of a vector, and \( |*| \) refers to an absolute value.

3. EXPERIMENTAL SETUP

Two types of experiment are conducted for intra-shape (between a main shape and its distorted versions) and inter-shape (between two main shapes) groups, namely 1) visual similarity (VS) test and 2) robustness test. The objectives of the experiments are to explore 1) how well the feature vector proposed can be used to distinguish shapes and 2) whether the visual similarity is preserved under random distortion conditions while using the shape feature proposed. In other words, if the similarity measure \( S \) satisfies the condition \( S(A, B) > S(A, C) \Leftrightarrow VS(A, B) > VS(A, C) \) for the given shapes A, B, C even under random distortion.
where parameter values are explained in Table 2.

The function derivation are

axis is formed in two ways as given in (2.1) - (2.3). They are

coefficients.

approach given in (2.2) for evaluation of parameters.

evaluation is achieved by similarity scores S1 and S2 given

figures to explore effect of each parameter set. Parameter

results are depicted with average % similarity scores in

developed. We proposed three approaches to derive a

were conducted to evaluate the performance of the approach

are 1) function value assigning, 2) making use of the slope

parameters S1 and S2 accomplish this requirement when used

and horse [3]. Experiments show that the similarity

similarity' may not be acknowledged by a 'visual similarity'
as in the computation of similarity between man, centaur,

A small shape database is created from three

cases are 1) function value assigning, 2) making use of the slope

coefficients. (2.3) approaches give similar results; see Figures 8-10 and

Third shape function derivation approaches given in the

the angle distance is larger than the magnitude distance.

significant change in computation of similarity score, see

produces a perfect recall and almost perfect precision on the

A) Evaluation of similarity measures: The main challenge in

measuring shape similarity is that a ‘computational

similarity’ may not be acknowledged by a ‘visual similarity’
as in the computation of similarity between man, centaur,

By an observation, while the distortion sensitivity increases,

that the angle distance is larger than the magnitude distance.

The scaling coefficient P5: It also does not show a

significant change in computation of similarity score, see

The experimenting with the weighting coefficient shows

the discriminative ability increases as well. The Results are

produced. A set of five parameters were experimented. They

are 1) function value assigning, 2) making use of the slope

function or not, 3) the order of edge and radial distance, 4)

x-axis scaling, and 5) feature vector size. In total 96

Feature vector size P4: The feature vector size

experimented are 10%, 25%, 50%, and 100% of the main polygons' vertices are distorted randomly.

A) Evaluation of similarity measures: The main challenge in

measuring shape similarity is that a ‘computational

similarity’ may not be acknowledged by a ‘visual similarity’
as in the computation of similarity between man, centaur,

and horse [3]. Experiments show that the similarity

measures S1 and S2 outperform S3 and S4. See

Figure 2 for comparison. As can be seen from Figure 2, by

setting the similarity threshold at 95%, using S1 and S2

produces a perfect recall and almost perfect precision on the

B) Evaluation of parameters:
The function type P1: ‘cs’ is more discriminative and its

sensitivity to distortion is similar to that of ‘or’, see Figure

3.
The slope of the function P2: The feature vector of the

function constructed from first derivative (P2 = ‘yes’) is

more discriminative and its sensitivity to distortion is similar
to P2 = ‘no’, see Figure 4.
The order of distance elements P3: The results are similar in

regard to the order of distance elements of the vector δ, see

Figure 5.
The size of a feature vector P4: It does not affect similarity

scores significantly, see Figure 6.
The scaling coefficient P5: It also does not show a

significant change in computation of similarity score, see

Figure 7.
The experimenting with the weighting coefficient shows

that the angle distance is larger than the magnitude distance.

As an observation, while the distortion sensitivity increases,

the discriminative ability increases as well. The Results are

summarized in Table 1.

Three shape function derivation approaches given in the

section 2.1 were experimented. The second (2.2) and third

(2.3) approaches give similar results; see Figures 8-10 and

Table 3.

5. CONCLUSION

In this study, a new shape feature which can be used to

match polygonal shapes with different number of vertices

was introduced. We proposed three approaches to derive a

shape function from which the shape feature is obtained. A

feature vector obtained from the Fourier coefficients of the

shape function is employed in the computation of the

similarity. The visual similarity and the robustness tests

were conducted to evaluate the performance of the approach

proposed. A set of five parameters were experimented. They

are 1) function value assigning, 2) making use of the slope

function or not, 3) the order of edge and radial distance, 4)

x-axis scaling, and 5) feature vector size. In total 96
experiments were conducted over each of 24 pairs of query and reference features. Overall, function type = ‘cs’ and use of slope = ‘yes’ outperform others while the parameters of order, scaling, and size are less sensitive to noise and closeness. Two types of similarity measures were employed; the first one is combination of angle and magnitude distances and the second group measures Euclidean distance between vectors compared. Experimental results show that the first group of similarity measures is robust and discriminative among similar and dissimilar shapes, whereas the second group does not perform well in discrimination when used with the shape feature proposed.

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6. REFERENCES


Table 1. Effect of parameters in similarity matching (√: strong, x: weak, ≈: similar, see Table 2 for explanation of parameter values)

<table>
<thead>
<tr>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>function type</td>
<td>use of slope</td>
<td>Order</td>
<td>% Length of feature vector</td>
<td>Scaling coefficient</td>
</tr>
<tr>
<td>or</td>
<td>cs</td>
<td>no</td>
<td>yes</td>
<td>Dr</td>
</tr>
<tr>
<td>Robustness</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Discrimination</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Table 2. Explanation of parameter values

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>or</td>
<td>use original vector $\tilde{y}$</td>
</tr>
<tr>
<td>cs</td>
<td>change sign of even indexed $\tilde{y}$ values, i.e. $\tilde{y}[2] = -\tilde{y}[2]$ and so on</td>
</tr>
<tr>
<td>yes</td>
<td>use slope function $\tilde{y}$ instead of $\tilde{y}$ itself; that is $\tilde{y} = \frac{y[1]-y[2]}{x[1]-x[2]}$</td>
</tr>
<tr>
<td>no</td>
<td>use $\tilde{y}$ itself</td>
</tr>
<tr>
<td>Dr</td>
<td>$\delta = (\text{edge distance}, \text{radial distance})$</td>
</tr>
<tr>
<td>rD</td>
<td>$\delta = (\text{radial distance}, \text{edge distance})$</td>
</tr>
</tbody>
</table>

Table 3. Experiment coding

<table>
<thead>
<tr>
<th>E-1</th>
<th>E-2</th>
<th>E-3</th>
<th>E-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>or</td>
<td>or</td>
<td>cs</td>
</tr>
<tr>
<td>P2</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>
Figure 4. Effect of using slope function (P2) on similarity scores

Figure 5. Effect of row order (P3) on similarity scores

Figure 6. Effect of size of feature vector (P4) on similarity scores

Figure 7. Effect of x-axis scale coefficient (P5) on similarity scores

Figure 8. Similarity scores by changing P1 and P2 using approach (2.1) (size = 25, \( c = 10 \), \( cA = 0.5 \), see Table 3)

Figure 9. Similarity scores by changing P1 and P2 using approach (2.2) (size = 25, \( c = 10 \), \( cA = 0.5 \), see Table 3)

Figure 10. Similarity scores by changing P1 and P2 using approach (2.3) (size = 25, \( c = 10 \), \( cA = 0.5 \), see Table 3)